Lorenz Gauge versus Coulomb Gauge: What is the Difference in Quantum Electromagnetics?

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Abstract— The quantization of electromagnetic field has been a subject of long studies [1–20]. This topic is important in macroscopic quantum optics/electromagnetics. Recently, we have quantized electromagnetics in the Lorenz gauge and in coordinate space [10], whereas most works have quantized them in Fourier space with Coulomb gauge. Depending on if the Lorenz gauge or Coulomb gauge is used in quantum electromagnetics, the commutator between the conjugate variable and the vector potential look very different. For the Coulomb gauge case, transverse delta function is involved, making the commutator complicated. For instance, the commutator in the Lorenz gauge is [10, Eqs. (93), (99)]

$$[\Pi_l(\mathbf{r}), A_j(\mathbf{r}')] = \frac{\hbar}{i}\delta_{ij}\delta(\mathbf{r}-\mathbf{r}')$$

where $\mathbf{A}$ is the vector potential, and $\mathbf{\Pi} = \dot{\mathbf{A}}$ is the conjugate variable. However, when Coulomb gauge is used, the above commutator becomes [5, 21]

$$[\Pi_l(\mathbf{r}), A_j(\mathbf{r}')] = \frac{\hbar}{i}\delta_{ij}^{tr}(\mathbf{r}-\mathbf{r}') = \frac{\hbar}{i}\left(\delta_{ij}\delta(\mathbf{r}-\mathbf{r}') + \frac{1}{4\pi}\frac{\partial^2}{\partial r_i \partial r_j}\frac{1}{|\mathbf{r}-\mathbf{r}'|}\right)$$

We have shown that classical Maxwell's equations can be derived from Hamiltonian theory [10, 13]. By invoking homomorphism between classical Hamiltonian theory and quantum Hamiltonian theory, the quantum Hamilton equations are homomorphic to classical Hamilton equations [10, 13, 16, 19, 20]. The commutator is important in demonstrating such homomorphism, and subsequently in the derivation of quantum Maxwell's equations. The commutator induces derivatives with respect to operators, which makes the derivation of quantum Maxwell's equations straightforward.

Also, by quantizing in the coordinate space, we can apply existing numerical methods for classical electromagnetics to solve quantum electromagnetics/optics problems [14]. Numerical methods will eventually help advancing the frontier of this field [23] as they have done in classical electromagnetics.

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